Born-Again Braneworld

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Abstract

We propose a cosmological braneworld scenario in which two branes collide and emerge as reborn branes with signs of tensions opposite to the original tensions of respective branes. In this scenario, the branes are assumed to be inflating. However, the whole dynamics is different from the usual inflation due to the non-trivial dynamics of the radion field. Transforming the conformal frame to the Einstein frame, this born-again scenario resembles the pre-big-bang scenario. Thus our scenario has features of both inflation and pre-big-bang scenarios. In particular, the gravitational waves produced from vacuum fluctuations will have a very blue spectrum, while the inflaton field will give rise to a standard scale-invariant spectrum.

1 Introduction

The inflationary universe scenario is a natural solution to fundamental problems of the big-bang model such as horizon problem. However, it is not a unique choice. A universe with an era of contraction is also a possibility. The pre-big-bang scenario is a realization of such a case in the superstring context[1]. Unfortunately, however, the pre-big-bang scenario suffers from a singularity problem which cannot be solved without understanding the stringy non-perturbative effects.

One of the remarkable features of superstring theory is the existence of extra dimensions. Conventionally, the extra dimensions are considered to be compactified to a small compact space of the Planck scale. However, recent revolutionary progress in string theory has lead to the brane-world picture [2]. In this talk, we consider a system of two branes having tensions of opposite signs, with the intermediate spacetime (bulk) described by an anti-de Sitter space (AdS_5) [3]. One of the branes is assumed to be our universe, and there exists an inflaton field which leads to inflation. The other brane is assumed to be vacuum but with a non-zero cosmological constant.

Assuming slow roll of the inflaton field, we may regard both branes as vacuum (de Sitter) branes. So far, mostly the static de-Sitter two-brane system is considered in the cosmological context. However, it is now well-known that a static de-Sitter two-brane system is unstable. We therefore investigate the non-trivial radion dynamics and dwell on its cosmological consequences [4].

2 Radion Dynamics

Most inflationary models are based on slow-roll inflation which have a sufficiently flat potential. In this section, we consider dynamics of branes with vacuum energy as a first order approximation of a slow-roll inflation model. Qualitative features of the brane cosmology can be understood by this simplified vacuum brane model.

We take the matter Lagrangeans to be $\mathcal{L}^{\oplus} = -\delta\sigma^{\oplus}$ and $\mathcal{L}^{\ominus} = -\delta\sigma^{\ominus}$. The effective action on the positive ($\oplus$) tension brane for this setup reads [5] (see also [6, 7, 8])

$$S_{\oplus} = \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-h}\left[\Psi R - \frac{3}{2(1-\Psi)}\Psi^{|\alpha}\Psi_{|\alpha}\right] - \delta\sigma^{\oplus}\int d^4x\sqrt{-h} - \delta\sigma^{\ominus}\int d^4x\sqrt{-h}\,(1-\Psi)^2\,. \qquad (1)$$

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Since our theory is a scalar-tensor type theory, we call this original action the Jordan-frame effective action. In order to discuss the dynamics of radion, it is convenient to move to the Einstein frame in which the action takes the canonical Einstein-scalar form. Applying a conformal transformation $h_{\mu\nu} = \frac{1}{\Psi} g_{\mu\nu}$ and introducing a new field $\eta = -\log\left|\frac{\sqrt{1-\Psi}-1}{\sqrt{1-\Psi}+1}\right|$, we obtain the Einstein-frame effective action

$$S_\oplus = \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-g}\left[R(g) - \frac{3}{2}\nabla^\alpha\eta\nabla_\alpha\eta\right] - \int d^4x\sqrt{-g}\ V(\eta)\ , \tag{2}$$

where ∇ denotes the covariant derivative with respect to the metric $g_{\mu\nu}$ and the radion potential now takes the form,

$$V(\eta) = \delta\sigma^\oplus\left[\ \cosh^4\frac{\eta}{2} + \beta\sinh^4\frac{\eta}{2}\ \right], \quad \beta = \frac{\delta\sigma^\ominus}{\delta\sigma^\oplus}\ . \tag{3}$$

We can also start from the effective action on the negative ($\ominus$) tension brane to obtain the same Einstein-frame effective action. By a conformal transformation $f_{\mu\nu} = \frac{1}{\Phi} g_{\mu\nu}$ and introducing a new field $\eta = -\log\left|\frac{\sqrt{\Phi+1}-1}{\sqrt{\Phi+1}+1}\right|$, we also arrive at Eq. (2).

We note that the two branes are infinitely separated when $\eta = 0$ ($\Psi = 1$) and they collide when $\eta = \infty$ ($\Psi = 0$). For definiteness, let us assume $\delta\sigma^\oplus > 0$. If $\delta\sigma^\oplus + \delta\sigma^\ominus > 0$, Ψ will move towards unity, i.e., the branes will move away from each other. While, if $\delta\sigma^\oplus + \delta\sigma^\ominus < 0$, the potential has a maximum at $\Psi_c = 1 + 1/\beta$, and the behavior depends on whether $\Psi > \Psi_c$ or $\Psi < \Psi_c$. If $\Psi > \Psi_c$, the branes will become infinitely separated. If $\Psi < \Psi_c$, the branes will approach to each other and eventually collide. From the 5-dimensional point of view, this is surely a singularity where the spacetime degenerates to 4-dimensions. However, as far as observers on the branes are concerned, nothing seems to go wrong. In fact, the action (1) is well-defined even in the limit $\Psi \to 0$. Let us assume that Ψ smoothly become negative after collision. Then replacing Ψ as $\Psi \to -\tilde{\Psi}$ in the action (1), we find

$$\begin{aligned} -S_\oplus &= \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-h}\left[\ \tilde{\Psi}R(h) + \frac{3}{2}\frac{1}{1+\tilde{\Psi}}\tilde{\Psi}^{|\alpha}\tilde{\Psi}_{|\alpha}\ \right] + \int d^4x\sqrt{-h}(-\mathcal{L}^\oplus) \\ &\quad + \int d^4x\sqrt{-h}\left(1+\tilde{\Psi}\right)^2(-\mathcal{L}^\ominus)\ . \end{aligned} \tag{4}$$

This is the same as the effective action on the negative tension brane

$$\begin{aligned} S_\ominus &= \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-f}\left[\Phi R(f) + \frac{3}{2(1+\Phi)}\Phi^{;\alpha}\Phi_{;\alpha}\right] \\ &\quad + \int d^4x\sqrt{-f}\mathcal{L}^\ominus + \int d^4x\sqrt{-f}\mathcal{L}^\oplus(1+\Phi)^2\ . \end{aligned} \tag{5}$$

except for the overall change of sign and the associated change of signs in front of the matter Lagrangeans. One can interpret this fact as follows. After collision, the positive tension brane turns into a negative tension brane together with the sign change of the matter Lagrangean, and vice versa for the initially negative tension brane. This implies that, if we live on either of the branes, our world transmutes into quite a different world and so does ourselves without much damage to the world. That is, we are born again!

3 Born-Again Braneworld

If our world had been initially a positive tension brane, we would be now on the negative tension brane. However, this case contradicts with observation. Therefore we take the position that we were initially on the negative tension brane before the collision.

Let us first investigate the cosmological evolution of the negative tension brane in the original Jordan frame. We consider the spatially isotropic and homogeneous metric on the brane,

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j\ , \tag{6}$$

where $a(t)$ is the scale factor and γ_{ij} is the metric of a maximally symmetric 3-space with comoving curvature $K = 0, \pm 1$. The field equations on the negative tension brane give

$$\dot{H} - \frac{K}{a^2} = -2\left(H^2 + \frac{K}{a^2}\right) - \frac{2\kappa^2}{3\ell}\delta\sigma^{\ominus} \ . \tag{7}$$

Integrating this equation, we obtain the Friedmann equation with a dark radiation,

$$H^2 + \frac{K}{a^2} = -\frac{\kappa^2}{3\ell}\delta\sigma^{\ominus} + \frac{C}{a^4} \ . \tag{8}$$

We also find the relation between the radion and the dark radiation,

$$\frac{\kappa^2\delta\sigma^{\ominus}}{3\ell}\frac{1+\Phi}{\Phi}\left[1 + \frac{(1+\Phi)}{\beta}\right] - H\frac{\dot{\Phi}}{\Phi} - \frac{1}{4}\frac{1}{1+\Phi}\frac{\dot{\Phi}^2}{\Phi} = \frac{C}{a^4} \ . \tag{9}$$

This gives, in particular, the relation between the initial conditions of the radion and the sign of the dark radiation.

To realize the born-again braneworld scenario, we consider a case of colliding branes. For simplicity, we assume $K = 0$. We can see numerically that Φ passes through zero smoothly and approaches -1, i.e., the reborn branes will be eventually infinitely separated.

Let us analyze this collision. We denote the Hubble constant at the time of collision $t = t_c$ by H_c. Applying Eq. (9) to the vicinity of the time of collision, we find

$$\Phi = -2(1-\sqrt{\gamma})H_c(t - t_c)\,; \quad \gamma = 1 - \frac{H_*^2}{H_c^2}\left(1 + \frac{1}{\beta}\right) \ , \tag{10}$$

where $H_*^2 = (\kappa^2/3\ell)(-\delta\sigma^{\ominus})$. As expected, Φ behaves perfectly smoothly around the time of collision. The brane geometry is, of course, perfectly regular as well. In fact, the Friedmann equation (8) continues to hold without a hint of collision.

Now we transform these quantities into the Einstein frame. Since $\Phi \to -1$ eventually, we may regard our present universe to be described by the Einstein frame. The relation between the Einstein frame and the Jordan frame is

$$\begin{aligned} ds_E^2 &= -dt_E^2 + b^2(t_E)\delta_{ij}dx^i dx^j \\ &= |\Phi|\left[-dt_J^2 + a(t_J)^2\delta_{ij}dx^i dx^j\right] , \end{aligned} \tag{11}$$

where we attached the subscripts E and J to the time coordinates to denote the cosmic time in the Einstein frame and the Jordan frame, respectively. Thus we have $b = \sqrt{|\Phi|}\,a, \quad dt_E = \sqrt{|\Phi|}\,dt_J$. Therefore, the Hubble parameter in the Einstein frame behaves in the vicinity of collision as

$$\frac{\dot{b}(t_E)}{b(t_E)} = \frac{1}{3t_E} + \frac{H_c}{\left(3(1-\sqrt{\gamma})H_c|t_E|\right)^{1/3}} \ , \tag{12}$$

where the collision time in the Einstein frame is set to be $t_E = 0$.

We note that in the Einstein frame, the universe is contracting rapidly just before the collision and the Hubble parameter diverges to minus infinitely at collision. Then the universe is reborn with an infinitely large Hubble parameter, which looks like a big-bang singularity. Thus, since there exists no singularity in the Jordan frame, the pre-big-bang phase and the post-big-bang phase in the Einstein frame is successfully connected. That is, our scenario is indeed a successful realization of the pre-big-bang scenario in the context of the braneworld.

4 Observational Implication

As we can see from Eq. (8), the universe will rapidly converge to the quasi-de-Sitter regime, while the radion can vary as far as the relation (9) is satisfied. In the Jordan frame, as the metric couples with the radion, the non-trivial evolution of the radion field affects the perturbations. This possibility discriminate our model from the usual inflationary scenario. On the other hand, the inflaton does not couple directly with the radion field. Hence, the inflaton fluctuations are expected to give adiabatic fluctuations with a flat spectrum. This feature is an advantage compared with the pre-big-bang model.

4.1 Radion fluctuation

To study the behavior of the radion fluctuations, it is convenient to work in the Einstein frame. The action for the curvature perturbation $\mathcal{R}$ on the $\delta\eta = 0$ (i.e., radion-comoving) slice reads

$$S = \frac{1}{2}\int d\eta\, d^3x\, z^2 \left[\mathcal{R}_c'^{\,2} - \mathcal{R}_c^{|i}\mathcal{R}_{c\,|i}\right] , \tag{13}$$

where $\mathcal{H} = b'/b$ and

$$\mathcal{R}_c = \mathcal{R} - \mathcal{H}\frac{\delta\eta}{\eta'} , \quad z = \sqrt{\frac{3\ell}{2\kappa^2}}\,\frac{b\eta'}{\mathcal{H}} . \tag{14}$$

As the background behaves as $b \sim (-\tau)^{1/2}$, $\mathcal{H} \sim (2\tau)^{-1}$ and $\eta' \sim (-\tau)^{-1}$, we have $z \propto b$, and the positive frequency modes for the adiabatic vacuum is given by

$$\mathcal{R}_{c,k} \sim \sqrt{\frac{\pi\kappa^2}{6H_*\ell}}\, H_0^{(1)}(-k\tau) , \tag{15}$$

where we have normalized b as $b = |H_*\tau|^{1/2}$. Then we have

$$\langle \mathcal{R}_c^2 \rangle_k \equiv \frac{k^3}{2\pi^2}P(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_{c,k}|^2 \sim \frac{k^3}{H_* M_{pl}^2} , \tag{16}$$

where $M_{pl}^2 = \kappa^2/\ell$. Thus the spectrum is very blue. If we define the spectral index by $P(k) \propto k^{n-4}$, this implies $n = 4$.

4.2 Gravitational waves

Next, consider the tensor perturbations:

$$ds^2 = b^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right] \tag{17}$$

where h_{ij} satisfy the transverse-traceless conditions, $h_{ij}{}^{,j} = h^i{}_i = 0$. As for the gravitational tensor perturbations, we have

$$h_k'' + 2\mathcal{H}h_k' + k^2 h_k = 0 , \tag{18}$$

where h_k is the amplitude of h_{ij}. Since $\mathcal{H} \sim (2\tau)^{-1}$, h_k has approximately the same spectrum as that for $\mathcal{R}_c$, including the magnitude. In particular, the spectral index for the gravitational waves is also $n = 4$ (with the spectral index defined by $P_h(k) \propto k^{n-4}$ as in the case of scalar curvature perturbation; for the tensor perturbation, the conventional definition is $n_T = n - 1$). Provided that inflation ends right after collision, this gives a sufficiently blue spectrum that can amplify Ω_g by several order of magnitudes or more on small scales as compared to conventional inflation models. Thus, there arises a possibility that it may be detected by a space laser interferometer for low frequency gravitational waves such as LISA [9].

4.3 Inflaton perturbation

To investigate the inflaton perturbation rigorously, one needs to introduce an inflaton field explicitly and consider a system of equations fully coupled with the radion and the metric perturbation. The estimation shows that the effect of the metric perturbation induced by radion fluctuations on the inflaton perturbation is small. Hence, the inflaton fluctuations will have a standard scale-invariant spectrum.

5 Conclusion

In this paper, we proposed a scenario in which two branes collide and are reborn as new branes, called the born-again braneworld scenario. Our model has the features of both inflationary and pre-big-bang

scenarios. In the original frame, which we call the Jordan frame since gravity on the brane is described by a scalar-tensor type theory, the brane universe is assumed to be inflating due to an inflaton potential. While, the radion, which represents the distance between the branes and which acts as a gravitational scalar on the branes, has non-trivial dynamics and theses vacuum branes can collide and pass through smoothly. After collision, it is found that the positive tension and the negative tension branes exchange their role. Then, they move away from each other, and the radion becomes trivial after a sufficient lapse of time. The gravity on the originally negative tension brane (whose tension becomes positive after collision) will then approach the conventional Einstein theory except for tiny Kaluza-Klein corrections.

One can also consider the cosmological evolution of the branes in the Einstein frame. Note that the two frames are indistinguishable at present if our universe is on the positive tension brane after collision. In the Einstein frame, the brane universe is contracting before the collision and one encounters a singularity at the collision point. This resembles the pre-big-bang scenario. Thus our scenario may be regarded as a non-singular realization of the pre-big-bang scenario in the braneworld context.

As our braneworld is inflating and the inflaton has essentially no coupling with the radion field, the adiabatic density perturbation with a flat spectrum is naturally realized. While, as the collision of branes mimics the pre-big-bang scenario, the primordial background gravitational waves with a very blue spectrum may be produced. This brings up the possibility that we may be able to see the collision epoch by a future gravitational wave detector such as LISA.

References

[1] M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993) [arXiv:hep-th/9211021].

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398];
P. Horava and E. Witten, Nucl. Phys. B **460**, 506 (1996) [arXiv:hep-th/9510209];
P. Horava and E. Witten, Nucl. Phys. B **475**, 94 (1996) [arXiv:hep-th/9603142].

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[4] S. Kanno, M. Sasaki and J. Soda, "Born-again braneworld," to appear in Prog. Teor. Phys., arXiv:hep-th/0210250.

[5] S. Kanno and J. Soda, Phys. Rev. D **66**, 083506 (2002) [arXiv:hep-th/0207029].

[6] T. Wiseman, Class. Quant. Grav. **19**, 3083 (2002) [arXiv:hep-th/0201127].

[7] S. Kanno and J. Soda, Phys. Rev. D **66**, 043526 (2002) [arXiv:hep-th/0205188].

[8] S. Kanno and J. Soda, "Braneworld effective action at low energies," arXiv:gr-qc/0209087;
J. Soda and S. Kanno, "Holographic view of cosmological perturbations," arXiv:gr-qc/0209086;
T. Shiromizu and K. Koyama, "Low energy effective theory for two brane systems: Covariant curvature formulation," arXiv:hep-th/0210066.

[9] P. Bender et. al, LISA. Pre-Phase A Report, second edition, July 1998. A choice of reference texts on the LISA project can be found at http://www.lisa.uni-hannover.de/lisapub.html.